Rescheduling Rolling Stock Rosters as a Profit Optimization Problem

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Abstract

We address rescheduling rolling stock rosters with maintenance constraints as a profit maximisation problem with fixed prices. We propose an approximate solution method that combines in a new way existing techniques. Computational tests with data from a North American railway company done in the same conditions as in production environment show that considerable revenue growths and cost savings can be obtained by using this approach. Optimality gaps for small problem instances are also supplied.

Keywords: rolling stock rosters, rescheduling, profit optimization

1. Introduction

Rescheduling rolling stock rosters (hereafter rosters}) or circulations is usually regarded in the literature as a cost minimisation problem with the passenger demand being modelled as a hard or soft constraint ([2]). This approach is not suitable for railway operators that run trains with reserved seats because in this context the maximisation of profit cannot be achieved just by minimising costs, it should also consider revenue. Some authors considered both aspects, but with limitations. For instance, [5] proposes a model that processes requests for extra train capacity from the commercial department trying to satisfy them as much as possible. More recently [3] proposed a model for handling contingency scenarios that considers both cost and revenue but without considering maintenance constraints.

In order to overcome these limitations, we propose an hybrid approach for solving approximately the Maximum Profit Rolling Stock Rescheduling (MPRSR) problem with maintenance constraints assuming fixed ticket prices. We describe the approach and how computational tests with data from a North American Railway Company (hereafter NARC) show its effectiveness in increasing expected profits. These tests were performed in the same conditions as in the production environment of NARC, in the sense that they were performed not only with NARC's data but with our algorithmic approach fully integrated in a future release of the software used by NARC to produce and adjust the operational schedules of its own rolling stock, which is based on FLEET, a standard decision-support product that creates and manages optimised vehicle schedules [6].

The MPRSR problem can be stated in the following way: given an original roster defined for calendar dates, given a set of updates in the passenger demands (for business and economy) of train trips as well as other updates (e.g. in the timetable of the trips), find, for a given planning period, the most profitable rosters that are identical (similar) to the original ones outside (inside) the planning period, that comply with all updates and that satisfy all operational constraints, namely maintenance constraints and many others (see [5] and [2]). The overall profit is equal to the revenue minus the operational cost, where revenue is derived from the expected tickets sold (obtained by matching the updated demand with train capacity for economy and business class), and cost includes aspects like track occupation, fleet depreciation, maintenance and energy consumption and crew utilisation.

In the context of NARC the rosters comply with a special requirement. The composition assigned to each trip has two parts: one, called the base composition, formed with vehicles that always circulate together as shown in Figure 1; and a second part formed with vehicles that don't have this restriction. The base composition is usually formed by a locomotive, a business and an economy carriage, but it can also include more economy carriages and a second locomotive in the rear.

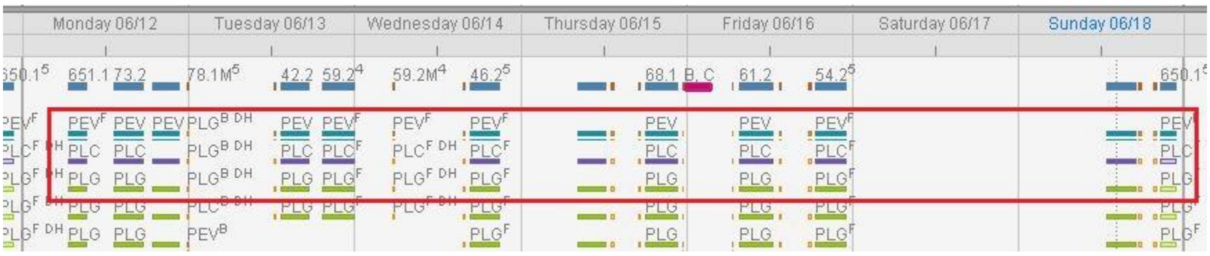

Figure 1: Locomotive roster showing the base composition where it belongs to (inside rectangle)

2. Solution method

Since the problem cannot be solved exactly, due to the size of NARC's problem instances, we propose an approximate solution method that is inspired on the concept of base composition described earlier.

The solution method solves the problem in two stages:

- produce rosters for the base composition,
- produce rosters for the extra carriages used to form compositions that are extensions of the base compositions obtained in the first stage.

In the *first stage* the following set covering problem with additional constraints is solved (using the notation adopted in [1]). We denote S as the set of original rosters. For every original roster $s \in S$, we consider the set J_s containing the possible final rosters that can replace the original roster s. To each roster j that can replace the original roster s we associate a cost c_{sj} that measures how undesirable it is to replace s by j. Finally, let T be the set of trips that need to be covered. For each trip $t \in T$ we define d_t as the cost of leaving t uncovered. We define a 0/1 parameter a_{sjt} that indicates whether the roster performs trip t or not. Furthermore, we introduce binary decisions variables x_{sj} that indicate whether a roster is selected or not to be part of the solution. We also introduce binary decisions variables (a.k.a slack variables) y_t that indicate whether a trip is left uncovered or not in the solution. With this notation the problem can be formulated in the following way:

$$\min \sum_{s \in S} \sum_{j \in J_s} c_{sj} x_{sj} + \sum_{t \in T} d_t y_t \tag{7}$$

$$s.t.$$

$$\sum_{s \in S} \sum_{j \in J_s} a_{sjt} x_{sj} + y_t \geq 1 \qquad \forall t \in T \tag{8}$$

$$\sum_{j \in J_s} x_{sj} = 1 \qquad \forall s \in S \tag{9}$$

$$x_{sj} \in \{0, 1\} \qquad \forall s \in S, j \in J_s \tag{10}$$

$$y_t \in \{0, 1\} \qquad \forall t \in T \tag{11}$$

The objective (7) is to minimize the total cost of the rosters and of the trips left uncovered. Constraints (8) ensure that a trip t is either performed or y_t is set to 1. Constraints (9) ensure that each original roster is replaced by one and only one final roster.

The cost c_{sj} is the weighted sum of the loss (cost minus revenue) observed in roster j and the deviation cost, i.e. amount of differences between final roster j and the original s. The cost d_t is set to be one order of magnitude larger than the upper bound of c_{sj}.

We solve the problem (7)-(11) with an heuristic based on the one described in [1], in the sense that it incorporates adjustments that aim at: (i) adapting the pricing procedure to produce feasible rosters, (ii) enforcing heuristically the satisfaction of constraints (9), and (iii) handling slack variables y_t.

The heuristic described in [1] runs an initial procedure where an initial set of columns is generated (during the execution of a simple loop) and then runs the main procedure where solutions are obtained during the execution of two embedded loops. In each iteration of each loop Lagrangian multipliers are obtained by solving the Lagrangian dual with the subgradient optimization method and columns (representing rosters) with negative reduced cost (computed with the Lagrangian multipliers) are generated with a dynamic programming procedure. In the outer and inner loops of the main procedure columns with lowest reduced cost are fixed as a way of intensifying the exploration of the search space. In the inner loop of the main procedure primal solutions are obtained with a greedy heuristic that selects columns starting with the ones with less reduced cost. The dynamic programming procedure computes shortest paths in a graph where: (i) distances measure reduced costs, (ii) nodes are trips and source and sink nodes related with the original roster and (iii) arcs, inside (outside) the planning period, connect trips that can be (that are) performed one after each other in the same final (original) roster. Paths with non-negative reduced cost or not complying with maintenance and other constraints are not included in the set of generated columns. All loops have upper bounds on the number of overall iterations and on the number of iterations without improvement.

We introduced the following modifications in the heuristic presented in [1]:

- the pricing procedure uses a labelling algorithm (see [4]) that solves a resource constrained shortest path problem where resources were set up properly to rule out paths in the trip graph, namely paths that correspond to rosters violating maintenance or any other relevant constraint;
- a procedure was introduced to repair and improve primal solutions obtained by the greedy heuristic; for the original rosters that have more than two final rosters (cases where (9) is not satisfied), it keeps one and sets each trip covered by the other rosters as uncovered (by generating the corresponding slack variables); after that, it runs a steepest descent hill climbing search where neighbours are alternative ways of planning uncovered trips (i.e. instantiating the corresponding slack variables with value 0), which can be: (i) inserting it in a normal roster or (ii) replacing it with another trip (overcovered or not); at each iteration the cheapest neighbour, according to cost function (1), is chosen; the process runs until it reaches a point where no neighbours are found.

The *second stage* is solved with the approach described in [2], which uses an hypergraph network model. In our case the hypergraph only contains hyperarcs related with the extra carriages that are used to enlarge the compositions obtained in the first stage. This reduces considerably the overall number of hyperarcs to a number that makes the problem tractable. In order to maximise profit this model favours (avoids) the assignment of extra carriages to trips with lack (excess) of seats.

Remark: by reducing the base composition to a single vehicle, e.g. a locomotive, this approach can be used in railway operators that don't use the concept of base composition.

3. Computational results

We made computational tests in the same conditions as in the production environment of NARC (e.g. same planning software, business rules, data, etc.). We created the problem instance groups shown in Table 1, representing different rescheduling scenarios over rosters of more than 150 rolling stock units (created based on real data from NARC) over planning periods that go from 1 to 5 days. Each instance group contains several versions of the same rescheduling scenario where the passenger demand of the train trips was randomly perturbed in 10 to 50 different ways.

Maintenance constraints adopted by NARC define a maintenance of type B (C) performed every week (two weeks), with the particularity that maintenance C includes B.

Problem instance group	**Planning period [days]**	**Number of trips**	**Average number of trips with changed demand [%]**	**Average demand increase [%]**	**Average demand decrease [%]**
P1	1	136	40.15	14.23	14.59
P2	1	135	41.48	14.24	14.65
P3	2	208	44.71	14.10	14.40
P4	4	327	45.57	14.26	14.55
P5	5	403	46.15	14.26	14.64

Table 1: Characterization of problem instances

For each instance group we present in Table 2 two average results, the first (second) obtained by solving the instances as a pure profit maximization (as a mixed profit maximization and deviation minimization) problem. The first (second) result is more suitable to be used before (after) tickets start being sold. As shown, there is a clear trade-off between the amount of changes allowed in the solution and the profit increase with respect to the original solution. For the smaller instances we were able to solve the problem with an exact approach (namely [2]), which allowed us to compute the optimality gap shown. For the larger instances the exact approach runs out of memory.

Problem instance group	**Average profit increase [%]**	**Average optimality gap [%]**	**Average changed compositions [%]**	**Number of maintenances of type B**	**Number of maintenances of type C**
P1	0.61 0.47	2.41	9.65 1.46	56	51
P2	0.24 0.08	2.47	3.05 0.42	52	52
P3	0.48 0.45	-	7.56 4.78	89	89
P4	0.084 0.011	-	5.10 0.15	168	156
P5	1.23 0.23	-	61.14 2.99	200	188

Table 2: Computational results

The profit increase values, shown in Table 2, are given in percentage terms and may seem small, but, if we translate them into monetary units, they correspond to potential annual profit growths that can go up to 3 million dollars.

4. Conclusions and future work

We addressed a fleet rescheduling problem that is relevant for train operations with reserved seats. Because practical problem instances cannot be solved exactly we proposed an approximate solution method that combines existing techniques in a new way. By using this approach in the same conditions as in production environment of a North American railway company we were able to obtain, in rescheduling scenarios with changes in the passenger demand, consistent profit growths that are larger when more changes in the original solution are allowed. We also reported optimality gaps for small problem instances.

Based on these promising results, we expect soon to deploy our approach to the above mentioned North American railway company, so that it can be used in practice. As our approach is fully integrated with the software tool used in production, this deployment is foreseen to be straightforward.

We also aim at extending the current approach with the ability to explore a larger space of final rosters, which is achieved by not forcing rosters to start and end in the same original roster, during the first stage of the solution method. By doing so, we expect to obtain solutions that are even more profitable.

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